Polarization Diffusion from Spacetime Uncertainty

Carlo R. Contaldi, Fay Dowker and Lydia Philpott
Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.

A model of Lorentz invariant random fluctuations in photon polarization is presented. The effects are frequency dependent and affect the polarization of photons as they propagate through space. We test for this effect by confronting the model with the latest measurements of polarization of Cosmic Microwave Background (CMB) photons.

All approaches to the problem of quantum gravity predict that the spacetime itself will suffer from quantum uncertainty at Planckian scales. This basic idea is the inspiration behind many attempts to formulate phenomenological models of quantum gravitational effects with potentially observable consequences. To date, much of the effort spent on modeling the effect of spacetime fluctuations has produced Lorentz symmetry violating models. However, with constraints on violations of Lorentz symmetry becoming tighter all the time (see *e.g.* [1]) it is more important than ever to discover quantum gravity phenomenology that respects Lorentz symmetry.

That such models can exist has been demonstrated in [2] and [3]. That work was motivated by the causal set approach to quantum gravity [4–6] but the scheme is quite general and does not depend on any details of the underlying theory except that it should be Lorentz invariant. The basic idea is that certain dynamical quantities such as particle trajectories are subject to minute, quasi-local, random fluctuations due to the uncertainty in spacetime structure at the Planck scale. The model-building strategy is straightforward: identify a space of states for the system, work out how Lorentz transformations act and hence deduce the most general Lorentz invariant diffusion process on that space.

In the case of a massive point particle, the outcome of this strategy is an Ornstein-Uhlenbeck-type process in which the momentum of the particle undergoes Brownian motion on the mass shell in proper time [2]. For massless particles, the Lorentz symmetry restricts the process to be a one dimensional diffusion in energy – the particles always travel on the light cone – but a second independent parameter enters which governs a drift in energy [3].

In this paper we will apply the strategy described above to polarization degrees of freedom as suggested in [7]. We model a photon classically as a point particle with a spacetime position x^μ, null momentum $k^\mu = (k^0, \vec{k})$ and a polarization state to be identified. A more realistic description would use wave packets and an even better model would take account of the quantal nature of photons. For now we assume that this classical state is a good approximate description of each of the free streaming photons produced by astrophysical and cosmological sources which reach our detectors.

The state space for a classical photon is therefore $\mathbb{M}^4 \times \mathcal{H}^3_0 \times \mathcal{B}$ where $\mathbb{M}^4$ is 4 dimensional Minkowski spacetime, $\mathcal{H}^3_0$ is the 3 dimensional "cone" of future-pointing null 4-vectors, and $\mathcal{B}$ is the space of polarization states which we will see is the Bloch sphere.

The polarization state of a massless particle of momentum k^μ can be given by a complex 4-vector a^μ such that $k^\mu a_\mu = 0$ and $a^{\mu *} a_\mu = 1$. The vector $a'^\mu = a^\mu + \lambda k^\mu$, for any complex number λ, will describe the same state. To eliminate this gauge freedom, we can consider the polarization state to be given by the complex two form, $P = k \wedge a$, whose components, $P_{\mu\nu} = k_\mu a_\nu - a_\mu k_\nu$ satisfy the Lorentz invariant conditions

$$P^{\mu\nu} k_\nu = 0\,, \tag{1}$$
$$P^{\mu\nu} P_{\mu\nu} = 0\,, \tag{2}$$
$$P^{\mu\nu *} P_{\mu\nu} = 0\,, \tag{3}$$
$$P^{\mu\nu *} P_{\mu\sigma} = k^\mu k_\sigma\,. \tag{4}$$

If $k^\mu = s^\mu$ where $s^\mu := (1,0,0,1)$, P has the following components

$$P_{\mu\nu} = \begin{pmatrix} 0 & -a_1 & -a_2 & 0 \\ a_1 & 0 & 0 & -a_1 \\ a_2 & 0 & 0 & -a_2 \\ 0 & a_1 & a_2 & 0 \end{pmatrix}\,, \tag{5}$$

where a_1 and a_2 are complex numbers such that $|a_1|^2 + |a_2|^2 = 1$. This corresponds to a polarization vector $a_\mu = (0, a_1, a_2, 0)$.

The phase of the 2-d complex unit vector (a_1, a_2) is not relevant for the polarization state of a single photon and so the polarization state space has two real dimensions: it is the Bloch sphere, $\mathcal{B} \cong \mathbb{CP}^1$. Let α and β be, respectively, the usual polar and azimuthal angles on $\mathcal{B}$, then they are related to the components of $P_{\mu\nu}$ by

$$a_1 = \frac{e^{i\gamma}}{\sqrt{2}}\left(\cos\frac{\alpha}{2} + e^{i\beta}\sin\frac{\alpha}{2}\right)\,, \tag{6}$$
$$a_2 = i\frac{e^{i\gamma}}{\sqrt{2}}\left(\cos\frac{\alpha}{2} - e^{i\beta}\sin\frac{\alpha}{2}\right)\,, \tag{7}$$

where γ is an irrelevant phase. The north and south poles, $\alpha = 0, \pi$, are the circularly polarized states and the equator, $\alpha = \pi/2$, consists of the linearly polarized states.

Now consider a general photon state $(k^\mu, P_{\mu\nu})$. For a general k^μ, the polarization 2-form $P_{\mu\nu}$ must be transformed by a Lorentz transformation that takes k^μ to s^μ in order for it to be compared to the standard polarization basis and its coordinates on $\mathcal{B}$ determined. This can

be done using a *standard Lorentz transformation* defined for example in [8]. If $P(k)$ is the polarization 2-form thus transformed, then it will have components of the form (5) and

$$P(k)_{\mu\nu} = s_\mu a_\nu - a_\mu s_\nu \,, \tag{8}$$

where $a_\mu = (0, P(k)_{10}, P(k)_{20}, 0)$. The (α, β) coordinates of the polarization state on $\mathcal{B}$ are then obtained from (6) with $a_1 = P(k)_{10}$ and $a_2 = P(k)_{20}$.

In this way, every photon state is specified by coordinates $(x^\mu, k^\mu, \alpha, \beta)$ on $\mathbb{M}^4 \times \mathcal{H}_0^3 \times \mathcal{B}$.

Under a Lorentz transformation the photon state $(k^\mu, P_{\mu\nu})$ transforms in the usual way as a vector and 2-tensor and it can be shown that this translates into a *polar rotation* on $\mathcal{B}$, a rotation around the north-south polar axis generated by $\frac{\partial}{\partial\beta}$. Details of these derivations will appear elsewhere.

The Stokes parameters (see e.g. [9]) are a convenient way to parameterize the polarization of a beam of electromagnetic radiation. A monochromatic beam with Stokes parameters (I, Q, U, V) can be modeled as a bunch of photons with the same momentum k^μ and polarization states distributed over $\mathcal{B}$. I is the intensity of the beam and since our process preserves particle number I is fixed. If a beam consists of photons of momentum k^μ which are all in the same polarization state $(\alpha, \beta) \in \mathcal{B}$ then the Stokes parameters of this perfectly polarized beam are $Q = I \sin\alpha \cos\beta$, $U = I \sin\alpha \sin\beta$ and $V = I \cos\alpha$.

If the photons have a distribution of polarizations the Stokes parameters are weighted by the probability density $\rho(\alpha, \beta)$ on $\mathcal{B}$ *e.g.* $Q = I \int_{\mathcal{B}} \sin\alpha \cos\beta \; \rho(\alpha, \phi) \, d\alpha d\beta$ and similarly for U and V. There are many distributions that will model a given set of Stokes parameters. For example, an unpolarized beam, $Q = U = V = 0$, could be modeled by a uniform distribution of linearly polarized states, or a uniform distribution on the two circularly polarized states alone. In general, the more spread out the distribution on $\mathcal{B}$, the smaller the *polarization fraction*, $\mathcal{P} := \sqrt{Q^2 + U^2 + V^2}/I$.

Having identified the state space of the photon as $\mathbb{M}^4 \times \mathcal{H}_0^3 \times \mathcal{B}$ we can deduce the most general Lorentz invariant diffusion process on this space. As described in [3] the trajectory in spacetime is simple: the photon moves along null lines according to $\frac{dx^\mu}{d\lambda} = k^\mu$ where λ is affine time. Moreover the process on the momentum space $\mathcal{H}_0^3$ must not disturb the blackbody nature of the CMBR spectrum over the age of the universe. This means that we can neglect this effect for the purposes of this paper: we assume that the photon's frequency is constant along its worldline. We are left with the task of deducing the Lorentz invariant diffusion equation on $\mathcal{B}$.

We refer to coordinates on $\mathcal{B}$ as $X^A = (\alpha, \beta)$. Then, following [10], the most general diffusion equation on $\mathcal{B}$ is

$$\frac{\partial\rho}{\partial\lambda} = \partial_A \left(K^{AB} \, n \, \partial_B \left(\frac{\rho}{n} \right) - u^A \rho \right) \,, \tag{9}$$

where λ is affine time, K^{AB} is a symmetric, positive semi-definite 2-tensor, u^A is a vector and n is a scalar density ("density of states") on $\mathcal{B}$. These geometric quantities are the phenomenological parameters of the model and must be Lorentz invariant.

There is an embarrassment of choice of parameters because Lorentz transformations act as polar rotations only. Any tensor, vector or scalar density that does not depend on the azimuthal angle, β, is Lorentz invariant. Free parameters that are whole functions do not make for powerful phenomenology. If, however, we restrict attention to the *linear* polarization states alone (corresponding to setting Stokes parameter V to zero), the model recovers its predictive power.

The space of linearly polarized states is the unit circle, the equator of the Bloch sphere, and the Lorentz transformations act as rotations of the circle. The coordinate around the circle is β and there is, up to a constant factor, one Lorentz invariant vector, $\partial/\partial\beta$. A Lorentz invariant density n must be constant on the circle. We deduce a simple diffusion-cum-drift on the circle for the distribution $\rho = \rho(\alpha, \beta)$:

$$\frac{\partial\rho}{\partial\lambda} = c \frac{\partial^2}{\partial\beta^2} \rho - d \frac{\partial}{\partial\beta} \rho \,, \tag{10}$$

where $c > 0$ and d are constants.

Transforming from affine time to "cosmic time", *i.e.* time in the observatory frame, $t = h\nu\lambda$ where h is Planck's constant and ν is the frequency of the photon, we have

$$\frac{\partial\rho}{\partial t} = \frac{c}{\nu} \frac{\partial^2}{\partial\beta^2} \rho - \frac{d}{\nu} \frac{\partial}{\partial\beta} \rho \,. \tag{11}$$

We absorbed the h into the free parameters governing the diffusion and drift. We see that the rates of diffusion and drift in polarization angle are frequency dependent. Note that the Lorentz symmetry we have assumed is only invariance under the proper orthochronous component of the full Lorentz Group. The drift term explicitly breaks parity invariance.

Our model has assumed that spacetime is Minkowski spacetime. Following [3] we can model the effect of an expanding universe by setting the frequency to depend on time, $\nu = \nu(t)$, such that $a(t)\nu(t) = a_0\nu_0$ where $a(t)$ is the scale factor of the universe, a_0 is the current value of a and ν_0 is the current (observed) value of the frequency of the photon. If we define a new time coordinate t' by $dt'/dt = a(t)/a_0$ then our diffusion equation keeps the same form as (11)

$$\frac{\partial\rho}{\partial t'} = \frac{c}{\nu_0} \frac{\partial^2}{\partial\beta^2} \rho - \frac{d}{\nu_0} \frac{\partial}{\partial\beta} \rho \,. \tag{12}$$

For a matter dominated universe $a \sim t^{2/3}$ and a range of t of 10^{60} Planck times becomes a range of t' of $3/5 \times 10^{60}$ Planck times. We drop the subscript 0 from ν and the prime from t in what follows.

Consider a beam of photons of frequency ν whose polarization is initially described by Stokes parameters (U, Q). The drift will result, after a time t in a beam whose polarization angle, Φ, has rotated by $\chi := td/\nu$:

$$\Phi = \tan^{-1}\left(\frac{U}{Q}\right) \to \Phi' = \Phi + \chi\,. \tag{13}$$

The diffusion will result in a decrease in the magnitude of polarization by a factor $\exp(-\mu)$:

$$\mathcal{P} := \sqrt{U^2 + Q^2} \to \mathcal{P}' = e^{-\mu}\mathcal{P}\,. \tag{14}$$

where μ can be calculated to be $4tc/\nu$.

Clearly, such an effect would be most pronounced in photons that have propagated over long distances. Thus CMB photons, whose polarization is correlated over incoming directions and have traveled over cosmological distances without rescattering, offer the best chance to constrain the parameters in the model. The polarization of the photons is imprinted during the last stages of recombination as the photons decouple from baryons and enter the free streaming regime. The correlation in the polarization of photons arriving from different directions is encoded in a set of angular power spectra $C_\ell^{\rm XY}$ where XY are the different spectral cross-correlation in total intensity T and grad-type and curl-like components E and B respectively. The spectra are calculated by solving the full Einstein-Boltzmann system describing the evolution of perturbed fluids in a particular cosmological model [11]. The rotation and suppression of polarization along the trajectory will modify the angular power spectra of the observed CMB photons $C_\ell^{\rm XY} \to \tilde{C}_\ell^{\rm XY}$. The mapping for each spectrum is given by [12, 13]

$$\begin{aligned}
\tilde{C}_\ell^{EE} &= e^{-2\mu}\, C_\ell^{EE}\, \cos^2(2\chi)\,, \\
\tilde{C}_\ell^{BB} &= e^{-2\mu}\, C_\ell^{EE}\, \sin^2(2\chi)\,, \\
\tilde{C}_\ell^{TE} &= e^{-\mu}\, C_\ell^{TE}\, \cos(2\chi)\,, \\
\tilde{C}_\ell^{TB} &= e^{-\mu}\, C_\ell^{TB}\, \sin(2\chi)\,, \\
\tilde{C}_\ell^{EE} &= \frac{1}{2} e^{-2\mu}\, C_\ell^{EE}\, \sin(4\chi)\,,
\end{aligned} \tag{15}$$

where we have assumed no BB contribution to the original spectra (no primordial gravitational waves). The TT spectrum is not modified as it is not sensitive to the polarization of the photons.

A number of assumptions are implicit in the simple mapping given in (15). Firstly it assumes that the picture of recombination (when the polarization is imprinted on the CMB) and reionisation (a further source of polarization) is not altered in this model. It assumes the background cosmological evolution is the same for a given set of cosmological parameters. In fact, polarization on large scales is also generated after the universe is reionized and this could introduce a mild scale dependence of the diffusion-rotation effect.

To constrain this scenario with available CMB data

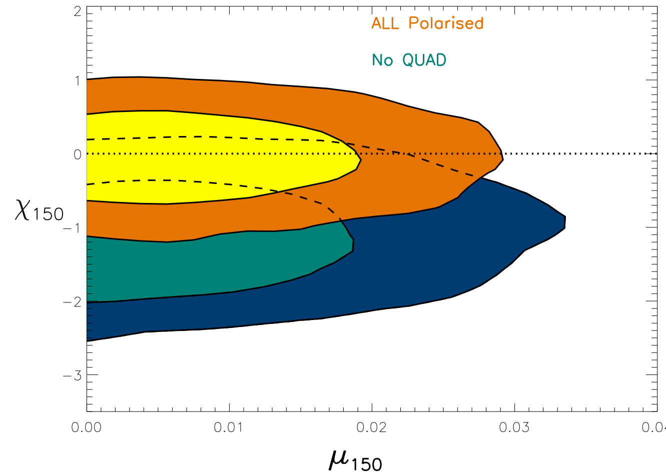

FIG. 1: The marginalized 2-d posterior density in the polarization rotation angle χ_{150} (in degrees) and polarization depth μ_{150} for the reference frequency of 150 GHz. The contours indicate the area bounding 68% and 95% of the density. We show the result for two data combinations; the first includes all polarization data and the second excludes the QUaD results. The results are consistent with no effect, however including the frequency dependence reduces the tension between the QuaD and 'no QUaD' combinations highlighted in [20].

we modify the `CosmoMC`[1] Monte Carlo Markov Chain (MCMC) package to fit for standard ΛCDM model parameters together with polarization rotation χ and polarization depth μ. The standard parameters are: cold dark matter and baryonic matter physical densities $\Omega_c h^2$ and $\Omega_b h^2$, angular diameter distance measure θ, optical depth to reionisation τ, and primordial scalar perturbation amplitude A_s and spectral index n_s. We assume a uniform prior of sufficient range in each parameter and do not include any primordial tensor contributions. However we *do* fit to TB, EB, and BB data since these are not expected to vanish any longer in the modified model.

We fit to a combination of data which includes all polarization sensitive experiments which have reported a detection of the EE power. These are the DASI results [14], the final CBIpol results [15], the Boomerang 2003 flight results [16], the WMAP 5-year results [17] and the latest BICEP [18] and QUaD [19] results. Of these, the last two contain the highest signal-to-noise determination of the polarization spectra on scales below a degree. Both BICEP and QUaD have published TB and EB data which are crucial in constraining polarization rotation effects [18, 20]. We include the published TB and EB band powers with band power window functions mimicking the published TE and EE ones. The frequency dependence

[1] http://cosmologist.info/cosmomc/

TABLE I: Marginalized 1-d constraints for the polarization rotation angle χ and 95% upper limits for the polarization depth μ.

	ALL Pol	No QuaD
Pol rotation: χ (degrees)	$-0.05^{+0.43}_{-0.43}$	$-1.11^{+0.55}_{-0.55}$
Pol depth: μ	$< 0.024(95\%)$	$< 0.026(95\%)$

is accounted for by scaling the effect for each experiment to a reference frequency of 150 GHz.

The MCMC chains sample the posterior density in the 8-dimensional parameter space. Once the sampling has converged we integrate the densities over the standard parameters which gives the marginalized posterior in χ_{150} and μ_{150}. We show the result of the marginalization in the χ_{150} μ_{150} plane in Fig. 1. We plot the 68% and 95% density contours for two data combinations. The tightest constraints are obtained by the combination of all polarized data and are consistent with no rotation and vanishing polarization depth. We also include the results for the case where the QUaD data is excluded. Although the 'no QUaD' result, driven mainly by the BICEP and WMAP measurements, prefers a non-zero rotation angle, the indication is weaker than that reported in [20]. However we *do* recover their result when not accounting for the frequency dependence. This may be an indication of a frequency dependent effect in the data whereby the lower frequency WMAP data tend favour less rotation given its effect is roughly twice as large compared to the reference frequency of 150GHz. Future observations at multiple frequencies will easily determine whether this is the case.

Table I shows the 1-d marginalized constraints on χ_{150} and μ_{150} showing the marginal bias towards a negative rotation driven mainly by the BICEP and WMAP data. The "All polarized" result is consistent with no effect.

The model building strategy employed here is based on the assumption that the fluctuations in dynamical variables due to spacetime uncertainty are small enough that they result, in the hydrodynamic approximation, in a continuous, Brownian motion through the state space. If spacetime uncertainty causes more violent, discontinuous jumps in the variables, this will have to be modeled by a Boltzmann equation rather than a diffusion equation. For this reason and also as data begins to constrain our phenomenology, the need for microscopic models that can give us a handle on the parameters from more fundamental physics becomes more acute. For example, an improvement on point particle models would be to treat particles as wave packets of a scalar field on a causal set using the discrete D'Alembertian operators described in [6, 7, 21]. Assuming continuity, however, the power of our model is its robustness: if photons can be modeled as classical particles with polarization degrees of freedom, then *any* Lorentz invariant effect of underlying spacetime uncertainty – whether due to discreteness, fluctuations, fuzziness, foaminess or whatever – will be of this form.

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